

# Mixing-induced CP violating sources for electroweak baryogenesis from a semiclassical approach

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## Abstract.

The effects of flavor mixing in electroweak baryogenesis is investigated in a generalized semiclassical WKB approach. Through calculating the nonadiabatic corrections to the particle currents it is shown that extra CP violation sources arise from the off-diagonal part of the equation of motion of particles moving inside the bubble wall. The mixing-induced source is of the first order in derivative expansion of the Higgs condensate, but is oscillation suppressed. The numerical importance of the mixing-induced source is discussed in the Minimal Supersymmetric Standard Model and compared with the source term induced by semiclassical force.

**Keywords:** Baryogenesis, CP violation

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Electroweak baryogenesis (EWBG) is a promising scenario for explaining baryon number asymmetry in the universe, unlike other scenarios valid at grand unification scale it can be tested by the upcoming collider experiments. An efficient way to generate large baryon number asymmetry is to generate it nonlocally, through a charge transportation mechanism [1]. In some models such as MSSM and the two-Higgs-doublet model, the bubble wall of the electroweak phase transition is typically thick [2, 3] for the particles giving dominant contributions, namely the particle typical Compton wave length  $\lambda \sim 1/T$  is much shorter than the wall width  $L_w \sim (10 - 20)/T$  where  $T$  is the critical temperature. In this regime the validity of semiclassical approach [4, 5, 6, 7, 8] should be justified for single flavor case, which provides an intuitively simple description by treating particle transportation as motion of WKB wave packages. In a slowly moving and CP violating Higgs condensate background, the dispersion relation for particles and antiparticles are modified differently, contributing to different semiclassical forces, which leads to a net excess or deficit of the particle number which can be converted into left-handed fermion number asymmetry. The asymmetry of the local fermion density then get transported in front of the bubble wall, which bias the baryon number violating processes.

The problem becomes more involved in the multiple flavor case, where the CP violating mass matrix has non-trivial space-time dependence. The CP violating effects may show up not only in the dispersion relations, but also in the mixing of states. In this work we generalize the semiclassical method by taking into account the non-adiabatic corrections from the spatially varying flavor mixings [9]. We restrict ourselves in the parameter

region where the mixing between the local mass eigenstates are relatively small, and can be treated as perturbations to the equation of motion for local mass eigenstates.

In two flavor mixing case the explicit form of the equation array for left-handed component can be rewritten as

$$\begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix} \begin{pmatrix} L_{s1} \\ L_{s2} \end{pmatrix} = 0, \quad (1)$$

with

$$\begin{aligned} D_{11} &= \omega^2 + \partial_z^2 - m_1^2 + 2\Sigma_{11}\partial_z + isA_{11}(\omega - is\partial_z) \\ D_{12} &= 2\Sigma_{12}\partial_z + isA_{12}(\omega - is\partial_z) \\ D_{21} &= 2\Sigma_{21}\partial_z + isA_{21}(\omega - is\partial_z) \\ D_{22} &= \omega^2 + \partial_z^2 - m_2^2 + 2\Sigma_{22}\partial_z + isA_{22}(\omega - is\partial_z). \end{aligned} \quad (2)$$

Since the off-diagonal elements  $D_{12(21)}$  contain differential operators, one can *not* obtain decoupled equations for  $L_{s1(2)}$  separately.

From the dispersion relation one can deduce the group velocity  $v_g \equiv (\partial\omega/\partial p_c)_z$  and semiclassical force  $F_i \equiv \omega(dv_g/dt)$  respectively

$$\begin{aligned} v_{gi} &= \frac{p_{0i}}{\omega} - s \frac{m_i^2 \text{Im} A_{ii}}{2\omega^2 p_{0i}}, \\ F_i &= -\frac{m_i m'_i}{\omega} - s \frac{(m_i^2 \text{Im} A_{ii})'}{2\omega^2}, \end{aligned} \quad (3)$$

where  $\omega$  is the energy of the state,  $\Sigma = U^\dagger \partial U$  and  $A = U^\dagger M \partial M^{-1} \partial U$  with  $M$  the mass matrix in flavor basis and  $U$  is the rotation matrix diagonalizing  $M^\dagger M$ .  $p_c$  is the canonical momentum and  $p_0^2 = \omega^2 - m^2$ . Note that only the second term in the force term is CP violating, and is proportional to the spin of the local mass states.

Taking the off-diagonal terms  $D_{12}$  and  $D_{21}$  as perturbations, the solutions can be written in a generic form

$$L_{si} = L_{si}^{(0)} + L_{si}^{(1)}, \quad (4)$$

where  $L_{si}^{(0)}$  is the lowest order solution satisfying

$$D_{ii}L_i^{(0)} = 0, \quad (5)$$

and  $L_{si}^{(1)}$  are the corrections due to the off-diagonal terms. The lowest order solution  $L_{si}^{(0)}$  is obtained by the usual WKB wave ansatz

$$L_{si}^{(0)} = w_i e^{i \int^z p_{ci}(z') dz'},$$

where  $p_{ci}$  is the canonical momentum and the function  $w_i$  provides the correct normalization for  $L_{si}^{(0)}$ .

Substituting the off-diagonal terms in to the equation, the first order perturbation takes the following form

$$L_{si} = L_{si}^{(0)} + L_{si}^{(1)} \simeq L_{si}^{(0)} + \varepsilon_i L_{si}^{(0)} \quad (6)$$

which are mixtures of the two unperturbed states. The mixing parameter for particle  $i$  to the first order of derivative is given by

$$\varepsilon_i = i \frac{2 \Sigma_{ij} p_{0j} + A_{ij} (s\omega + p_{0j})}{m_i^2 - m_j^2}, \quad (7)$$

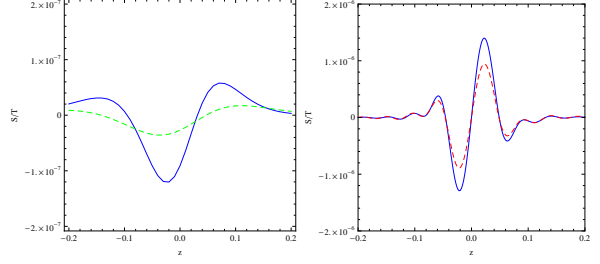
The mixing parameter for particle  $j$  can be simply obtained by replacing  $i \leftrightarrow j$  from the above expressions. It is clear that the expansion is only valid for  $\partial_z m_i / (\Delta m^2) \ll 1$ . The momentum dependencies comes from the differentiation operators in off-diagonal element  $D_{12}$ .

The CP violating force term, as it is proportional to spin  $s$ , only contribute to the spin-weighted density. To facilitate the comparison with the force term, we give the spin-weighted mixing-induced source term

$$\begin{aligned} S_{Li} &= \sum_s \frac{s}{2} \partial_\mu j_{Li}^{s\mu} \\ &= \frac{2m_i m_j}{m_i^2 - m_j^2} \left[ \text{Im} \Sigma g_L(p_{0i}, p_{0j}) - \frac{m_i}{m_j} \text{Im} \Pi_{ij} g_R(p_{0i}, p_{0j}) \right] \\ &\quad \cdot (p_{0i} - p_{0j}) \sin \int^z (p_{cj} - p_{ci}) dz', \end{aligned}$$

where  $\Pi$  is a similar quantity as  $\Sigma$  but for right-handed field. The functions  $g_{L,R}$  are given by

$$\begin{aligned} g_{L,R}(p_{0i}, p_{0j}) &= \frac{\omega \mp p_{0j}}{\sqrt{p_{0i} p_{0j} (\omega + p_{0i}) (\omega + p_{0j})}} \\ &\quad - \frac{\omega \pm p_{02}}{\sqrt{p_{0i} p_{0j} (\omega - p_{0i}) (\omega - p_{0j})}} \end{aligned}$$



**FIGURE 1.** (a) Left, the semiclassical force induced source term as a function of  $z$  ( $\text{GeV}^{-1}$ ). (b) Right, the mixing-induced source term as a function of  $z$ . The two curves in each plot corresponds to  $L_w = 10/T$  (solid) and  $15/T$  (dashed) respectively. The MSSM parameters are fixed at  $M_2 = 150$  GeV and  $\mu = 100$  GeV, with  $\phi_\mu = 0.02$ .

which is a momentum odd function.

Following the standard process to derive the transportation equations, one arrives at the usual form of the diffusion equation

$$-k_i \left( D_i \xi_i'' + v_w \xi_i' \right) + \tilde{\Gamma}_{ik}^d \sum_j \xi_j^{(k)} \simeq S_F + S_M \quad (8)$$

with  $\tilde{\Gamma}_{ik}^d = k_i \Gamma_{ik}^d$  the interaction rate.

The mixing and force induced source terms have the following form after thermal-average

$$S_F = - \frac{k_i v_w D_i}{T \left\langle \left( \frac{p_z}{\omega} \right)^2 \right\rangle} \left\langle \frac{p_z}{\omega} \delta F_{zi} \right\rangle', \text{ and } S_M = k_i \left\langle S_i^{pl} \right\rangle \quad (9)$$

where  $\xi \equiv \mu_i/T$  the rescaled chemical potential and  $D_i = \langle (p_z/\omega)^2 \rangle / \Gamma_i^d$  the diffusion constant.

In MSSM, the chargino transportation provides a dominant CP violating source to electroweak baryogenesis. The asymmetry in chargino number is converted into the asymmetry in left-handed top quarks through Yukawa interactions. The two Higgsino  $SU(2)$  doublets are  $\tilde{h}_1 = (\tilde{h}_{1L}^0, \tilde{h}_{1L}^-)^T$  and  $\tilde{h}_2 = (\tilde{h}_{2L}^0, \tilde{h}_{2L}^-)^T$  respectively. Together with the two charged gauginos  $\tilde{W}_L^+$  and  $\tilde{W}_L^-$ , the charginos are combined into two left- and right-handed four component spinors as  $\psi_L = (\tilde{W}_L^+, \tilde{h}_{2L}^+)^T$  and  $\psi_R = ((\tilde{W}_L^-)^c, (\tilde{h}_{1L}^-)^c)^T$ . The chargino mass term has the form  $\tilde{\psi}_R M(z) \psi_L$  in the wino-higgsino space with mass matrix

$$M(z) = \begin{pmatrix} M_2 & gH_2(z) \\ gH_1(z) & \mu \end{pmatrix}, \quad (10)$$

where  $M_2$  and  $\mu$  are soft supersymmetry breaking parameters containing CP phases and  $H_1(z)$  and  $H_2(z)$  the Higgs vacuum expectation values (VEVs).

In Fig.1, we give the source term calculated from Eq.(9) and rescaled by  $1/T$  from both semiclassical force and mixing-induced source term for wall width  $L_w = 10/T$  and  $15/T$  respectively. The MSSM softbreaking

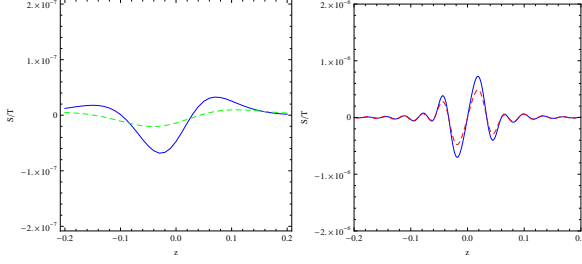


FIGURE 2. The same as Fig.1 with  $M_2 = 250$  GeV.

parameters are fixed at  $M_2 = 200$  GeV and  $|\mu| = 100$  GeV corresponding to a chargino mass differences  $m_1 - m_2 = 111$  GeV in the broken phase. The CP phase  $\phi_\mu$  is set to a typical value of  $\phi_\mu = 0.02$ . The curves given in Fig.1 show that both of source terms have nontrivial spatial dependencies. However, their origin are quite different. The variation of the force term comes from the third derivative of the kink-type Higgs condensate, which has one minimal and two maximums. While the mixing-induced source term varies due to both the wall profile variation and the oscillation. The oscillation leads to multiple local minimum appearing in the curve and is suppressed at large distance by the wall profile. For  $M_2$  around 200 GeV, the mixing induced source term peaks at  $z \simeq 0.03$  with an amplitude  $S_M/T \simeq 1.6 \times 10^{-6}$ , much larger than that from semiclassical force which peaks at  $z \simeq 0.08$  with  $S_F/T = 0.6 \times 10^{-7}$ . In Fig.2, we give the results for a larger  $M_2 = 250$  GeV, and still fix  $|\mu| = 100$  GeV, corresponding to a larger chargino mass differences  $m_1 - m_2 = 157$  GeV in the broken phase. One sees that for a larger chargino mass difference both the source terms becomes smaller as they are  $1/\Lambda$  suppressed. The oscillation in the mixing-induced source term becomes obvious and the wave length is shorter, thus the oscillation suppression is stronger. The mixing induced source term peaks at  $z \simeq 0.03$  with an amplitude  $S_M/T \simeq 8 \times 10^{-7}$ . Although significantly reduced, it still much larger than that from semiclassical force which peaks at  $z \simeq 0.08$  with  $S_F/T \simeq 4 \times 10^{-8}$ .

To estimate the oscillation suppression effects it is useful to define an averaged source over the wall width

$$\bar{S}_{M(F)} \equiv \frac{1}{TL_w} \int_0^{L_w} S_{M(F)}(z) dz. \quad (11)$$

We calculate the averaged source for different  $M_2 = 200 \sim 500$  GeV and list the results in Tab.I in Ref.[9] The results show that  $\bar{S}_M$  dominates over  $\bar{S}_F$  in the range  $200 \text{ GeV} \lesssim M_2 \lesssim 350 \text{ GeV}$ . With the value of  $M_2$  increasing, the averaged source term  $\bar{S}_M$  drops rapidly. For  $L_w = 10/T$ , at  $M_2 = 350$  GeV, the mixing induced source is only 5.4% of that at  $M_2 = 200$  GeV, the suppression is due to the increased oscillation frequency. The suppression in force term  $\bar{S}_F$  is mainly from the factor  $1/\Lambda$ ,

which makes it decrease slowly. At  $M_2 = 350$  GeV, it is still about 30% as large as that at  $M_2 = 200$  GeV. At  $M_2 = 200$  GeV, their relative size is  $\bar{S}_M/\bar{S}_F = 14.6$ . For large  $M_2 = 350$  GeV, although they are close in size, the mixing source still dominates with  $\bar{S}_M/\bar{S}_F = 2.63$ . This dominance has a mild dependence on the wall width. For  $L_w = 15/T$ , the relative size between the two kind of sources remains roughly the same, although both of the source term becomes smaller. The mixing-induced source (semiclassical force) term is  $\sim 40\%$  ( $\sim 30\%$ ) of that at  $L_w = 10/T$ . When the value of  $M_2$  is around 450 GeV, the two type of source term becomes comparable in size. For a very large  $M_2 = 500$  GeV, the semiclassical force term becomes dominate. The mixing-induced source term is about an order of magnitude smaller.

In conclusion, we have studied the effects of flavor mixing in a generalized WKB approach in which the off-diagonal terms in the equation of motion are taken into account as perturbations. With the presence of a slowly moving CP violating bubble wall, an extra mixing-induced CP violating source appears which exhibit an oscillation behavior in analogy to the neutrino mixings. We have made a numerical study of the oscillation suppression effects for chargino case in MSSM and shown that for a light  $200 \lesssim M_2 \lesssim 350$  GeV even in the small mixing case the mixing-induced source already indicate that a significant enhancement of the final baryon number asymmetry is possible, which will make MSSM a more realistic model for EWBG.

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